

$\rho \rightarrow 4\pi$ DECAY

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Abstract

The decay modes $\rho^0 \rightarrow 2\pi^+2\pi^-$ and $\rho^0 \rightarrow 2\pi^0\pi^+\pi^-$ are considered in the framework of the low energy effective chiral Lagrangian. The obtained values of the decay widths $\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-) = (16 \pm 1)\text{keV}$ and $\Gamma(\rho^0 \rightarrow 2\pi^0\pi^+\pi^-) = (6.0 \pm 0.2)\text{keV}$ do not contradict the existing upper limits and seem to be big enough for the corresponding processes to be observed in future high luminosity experiments.

The new generation experiments at the Novosibirsk e^+e^- collider VEPP-2M with two modern detectors [1, 2], as well as the planned experiments at the Frascati ϕ -factory DAΦNE [3] allow rare decays of the light vector mesons to be studied. This paper presents results of the calculation for the decay widths $\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-)$ and $\Gamma(\rho^0 \rightarrow 2\pi^0\pi^+\pi^-)$. The experimental search for these decays gave only upper limits

$$\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-) < 30 \text{ keV} [4]$$

and

$$\Gamma(\rho^0 \rightarrow 2\pi^0\pi^+\pi^-) < 6 \text{ keV} [5].$$

The significant increase of the total number of ρ -mesons expected in the new experiments mentioned above motivated this calculation.

Earlier theoretical studies of these decays assumed quasitwoparticle intermediate states. Renard calculated the width of the decay $\rho^0 \rightarrow 2\pi^0\pi^+\pi^-$ via the $\omega\pi$ intermediate state and obtained the width of 0.9 keV [6] well below the existing limit, while in [7] the value $\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-) = 172 \text{ keV}$ considerably higher than the existing limit was obtained assuming the $A_1\pi$ and $A_2\pi$ intermediate states. The recent work [8] pointed to the necessity of taking into account other intermediate mechanisms within the chiral model and presented the value of the width of $\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-)$. Particularly, they showed that in one of the Yang-Mills type models (the "Massive Yang-Mills approach" [9]) the value of 60 keV is obtained in obvious contradiction to the experimental limit. They also presented the results for two other models (the Hidden symmetry scheme and the naive Vector Dominance Model) which are 7.5 and 25 keV respectively.

However, it is well known [10, 11] that the simple version of the chiral Yang-Mills Lagrangian as in [9] should be corrected by special terms so that vector mesons can be naturally introduced into the Lagrangian without violating low-energy theorems of the

current algebra. In the present paper such a corrected lagrangian is used [12, 13, 14] to calculate the ρ -meson decay widths in both channels.

The process $\rho \rightarrow 4\pi$ is in general described by six classes of Feynman diagrams shown in Fig. 1. The square of the corresponding matrix element averaged over spin states is given by the following formula:

$$|\overline{M}|^2 = \frac{1}{3} g_{\rho\pi\pi}^2 \vec{J}^*(\rho \rightarrow 4\pi) \cdot \vec{J}(\rho \rightarrow 4\pi) \quad , \quad (1)$$

where $g_{\rho\pi\pi} J_\mu(\rho \rightarrow 4\pi)$ is the conserved current which describes the $\rho \rightarrow 4\pi$ transition.

Fig. 1. Feynman diagrams describing the $\rho \rightarrow 4\pi$ decay.

For the reaction $\rho^0(q) \rightarrow \pi^+(q_1) + \pi^+(q_2) + \pi^-(q_3) + \pi^-(q_4)$ only diagrams a-d from Fig. 1 contribute. The corresponding current has a form:

$$\begin{aligned} J_\mu^{\rho^0 \rightarrow 2\pi^+ 2\pi^-} = & \left(\frac{1}{3} - \alpha_k \right) \frac{1}{f_\pi^2} [6(q_1 + q_2 - q_3 - q_4)_\mu \\ & + (6q_3 \cdot q_4 + 2m^2) \left(\frac{(q - 2q_1)_\mu}{(q - q_1)^2 - m^2} + \frac{(q - 2q_2)_\mu}{(q - q_2)^2 - m^2} \right) \\ & - (6q_1 \cdot q_2 + 2m^2) \left(\frac{(q - 2q_3)_\mu}{(q - q_3)^2 - m^2} + \frac{(q - 2q_4)_\mu}{(q - q_4)^2 - m^2} \right)] \\ & + 2(1 + P_{12})(1 + P_{34}) \frac{g_{\rho\pi\pi}^2 ((q_2 + q_4)^2 - m_\rho^2 - im_\rho \Gamma_\rho)}{((q_2 + q_4)^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} \\ & \times \left[(q_4 - q_2)_\mu + \frac{q_1 \cdot (q_2 - q_4)}{(q - q_3)^2 - m^2} (q - 2q_3)_\mu + \frac{q_3 \cdot (q_2 - q_4)}{(q - q_1)^2 - m^2} (q - 2q_1)_\mu \right], \quad (2) \end{aligned}$$

where $m^2 = m_{\pi^\pm}^2 = q_i^2$, $\alpha_k = \frac{f_\pi^2 g_{\rho\pi\pi}^2}{m_\rho^2} \approx 0.55$, and P_{12} and P_{34} operators stand for the interchange of the momenta of the corresponding identical mesons.

For the process $\rho^0(q) \rightarrow \pi^+(q_+) + \pi^-(q_-) + \pi^0(q_1) + \pi^0(q_2)$ all six classes of diagrams contribute. One of them (f) contains the $\omega\pi$ intermediate state and is due to the anomalous part of the chiral Lagrangian.

The corresponding current J_μ can be presented as a sum of three terms, each of them representing a gauge invariant subset of diagrams ($J_\mu^{(i)} q^\mu = 0$):

$$J_\mu^{\rho^0 \rightarrow 2\pi^0 \pi^+ \pi^-} = J_\mu^{(1)} + J_\mu^{(2)} + J_\mu^{(3)}. \quad (3)$$

Diagrams of type a,b of Fig. 1 give after some algebraic transformations:

$$J_\mu^{(1)} = \left(\frac{1}{3} - \alpha_k\right) \frac{1}{f_\pi^2} (6q_1 \cdot q_2 + 2m_{\pi^0}^2) \times \left[\frac{(q - 2q_-)_\mu}{(q - q_-)^2 - m_{\pi^\pm}^2} - \frac{(q - 2q_+)_\mu}{(q - q_+)^2 - m_{\pi^\pm}^2} \right]. \quad (4)$$

The second piece arises from diagrams of type c,d,e of Fig. 1 and has the form

$$J_\mu^{(2)} = -g_{\rho\pi\pi}^2 (1 + P_{12}) \left\{ -\frac{1}{r_+ r_-} [2(q_+ - q_1)_\mu q \cdot (q_- - q_2) - 2(q_- - q_2)_\mu q \cdot (q_+ - q_1) + (q_2 + q_- - q_1 - q_+)_\mu (q_+ - q_1) \cdot (q_- - q_2)] \right. \\ \left. + \frac{1}{r_+} \left[(q_+ - q_1)_\mu - 2q_2 \cdot (q_+ - q_1) \frac{(q - 2q_-)_\mu}{(q - q_-)^2 - m_{\pi^\pm}^2} \right] - \frac{1}{r_-} \left[(q_- - q_2)_\mu - 2q_1 \cdot (q_- - q_2) \frac{(q - 2q_+)_\mu}{(q - q_+)^2 - m_{\pi^\pm}^2} \right] \right\}, \quad (5)$$

$$r_+ = (q_+ + q_1)^2 - m_\rho^2 + im_\rho \Gamma_\rho; \\ r_- = (q_- + q_2)^2 - m_\rho^2 + im_\rho \Gamma_\rho.$$

Finally, the third part of the current is determined by two diagrams of type f of Fig. 1 with the ω -meson intermediate state (note that the $\omega \rightarrow 3\pi$ effective vertex contains contributions from the contact term as well as from the $\rho\pi$ intermediate states):

$$J_\mu^{(3)} = \frac{3g_{\rho\pi\pi}}{8\pi^2 f_\pi} (1 + P_{12}) P_\mu \frac{F_1}{r_1}, \quad (6)$$

where

$$P_\mu = q_1 \cdot q_2 (q_{+\mu} q \cdot q_- - q_{-\mu} q \cdot q_+) + q_- \cdot q_2 (q_{1\mu} q \cdot q_+ - q_{+\mu} q \cdot q_1) \\ + q_+ \cdot q_2 (q_{-\mu} q \cdot q_1 - q_{1\mu} q \cdot q_-), \quad (7)$$

and

$$r_1 = (q - q_2)^2 - m_\omega^2 + im_\omega \Gamma_\omega, \quad (8) \\ F_1 = \frac{3g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} \left[1 - 3\alpha_k - \alpha_k \left(\frac{m_\rho^2}{r_{+-}} + \frac{m_\rho^2}{r_{+1}} + \frac{m_\rho^2}{r_{-1}} \right) \right], \\ r_{+-} = (q_+ + q_-)^2 - m_\rho^2 + im_\rho \Gamma_\rho, \\ r_{+1} = (q_+ + q_1)^2 - m_\rho^2 + im_\rho \Gamma_\rho, \\ r_{-1} = (q_1 + q_-)^2 - m_\rho^2 + im_\rho \Gamma_\rho.$$

The expressions for the current above use the values of the constants from [13].

The width of the decay is given by the following expression

$$\Gamma = \frac{N}{2m_\rho (2\pi)^8} R$$

where a factor N takes into account the identity of the final pions and equals $1/4$ for $2\pi^+2\pi^-$ and $1/2$ for $2\pi^0\pi^+\pi^-$. R is the above matrix element squared (formula (1)) integrated over the phase space of final particles. This integration was performed by two independent methods. In one of them the quantity R was represented as the following five-dimensional integral [15] and was calculated numerically:

$$R = \frac{\pi^2}{8m_\rho^2} \int_{s_1^-}^{s_1^+} ds_1 \int_{s_2^-}^{s_2^+} ds_2 \int_{u_1^-}^{u_1^+} \frac{du_1}{\sqrt{\lambda(m_\rho^2, s_2, s_2')}} \\ \times \int_{u_2^-}^{u_2^+} du_2 \int_{-1}^1 \frac{d\zeta}{\sqrt{1-\zeta^2}} |M|^2.$$

Here we introduced Kumar's invariant variables

$$s_1 = (q - q_1)^2, \quad s_2 = (q - q_1 - q_2)^2, \quad u_1 = (q - q_2)^2, \\ u_2 = (q - q_3)^2, \quad t_2 = (q - q_2 - q_3)^2,$$

$s_2' = s_2 + s + m_1^2 + m_2^2 - u_1 - s_1$ and $\arccos\zeta$ is an angle between $(\vec{q}_2, \vec{q}_1 + \vec{q}_2)$ and $(\vec{q}_3, \vec{q}_1 + \vec{q}_2)$ planes. $\lambda(x, y, z) = (x + y - z)^2 - 4xy$ is a conventional triangle function. The relation between t_2 and ζ as well as the expressions for the integration limits can be found in [15].

Another method used the Monte-Carlo procedure of the random star generation suggested by Kopylov [16]. Both methods gave consistent results.

The values of the widths obtained were

$$\Gamma(\rho^0 \rightarrow 2\pi^+2\pi^-) = (16 \pm 1) \text{ keV}$$

$$\Gamma(\rho^0 \rightarrow \pi^+\pi^-2\pi^0) = (6.0 \pm 0.2) \text{ keV}$$

and are close to the existing upper limits but do not contradict them.

From these results one can estimate the peak values of the cross section of four pion production in e^+e^- annihilation in the vicinity of the ρ -meson resonance. The obtained values are 0.12nb and 0.04nb respectively and give a real chance of observing these processes in the forthcoming experiments.

We have also calculated the width of a similar decay $\phi \rightarrow \eta\pi^+\pi^-\pi^0$. Unfortunately, the value of the width obtained corresponds to a very small branching ratio of about 10^{-11} and can hardly be observed in the near future.

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